UNIT 1
Expressions and the Number System

Study Guide Review

Vocabulary Development

Integrating the ELPS
Encourage English learners to refer to their notes and the illustrated, bilingual glossary as they review the unit content.

ELPS c.4.E  Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

MODULE 1  Real Numbers

8.2.A, 8.2.B, 8.2.D

Key Concepts
- An irrational number is a number that is not rational and cannot be written in the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \neq 0 \). (Lesson 1.1)
- The square root of a number is the number that when multiplied by itself has the original number as the product. Every positive number has a positive and negative square root. (Lesson 1.1)
- The set of real numbers consists of the set of rational numbers and the set of irrational numbers. (Lesson 1.2)
- Between any two real numbers is another real number. (Lesson 1.3)

Additional Resources
Assessment Resources
- Leveled Unit Tests: A, B, C, D
- Performance Assessment

Personal Math Trainer
Online Assessment and Intervention

Assessment Resources
- Leveled Unit Tests: A, B, C, D
- Performance Assessment
MODULE 2 Scientific Notation

Key Concepts
- Scientific notation is a method of expressing very large and very small numbers as a product of a number greater than or equal to 1 and less than 10, and a power of 10. *(Lesson 2.1)*
- To multiply a number by 10, move the decimal one place to the right, and to divide a number by 10, move the decimal one place to the left. *(Lessons 2.1, 2.2)*
EXAMPLE 1
Estimate the value of \( \sqrt{5} \), and estimate the position of \( \sqrt{5} \) on a number line.

\( \sqrt{5} \) is between the perfect squares 4 and 9. 
4 < \( \sqrt{5} \) < 9

Take the square root of each number. 
\( \sqrt{4} < \sqrt{5} < \sqrt{9} \)

\( 2 < \sqrt{5} < 3 \)

\( 2.2^2 = 4.84 \)
\( 2.3^2 = 5.29 \)

\( \sqrt{5} \) is between 2.2 and 2.3.

A good estimate is 2.25.

EXAMPLE 2
Write all names that apply to each number.

A. \( 5.\overline{3} \)
   rational, real

B. \( \frac{8}{3} \)
   whole, integer, rational, real

C. \( \sqrt{13} \)
   irrational, real

EXAMPLE 3
Order 6, \( 2\pi \), and \( \sqrt{38} \) from least to greatest.

\( 2\pi \) is approximately equal to 6.28.

\( \sqrt{38} \) is approximately 6.15.

\( \sqrt{36} < \sqrt{38} < \sqrt{49} \)
6 < \( \sqrt{38} \) < 7

6.1^2 = 37.21
6.2^2 = 38.44

\( 6 < \sqrt{38} < 7 \)

From least to greatest, the numbers are 6, \( \sqrt{38} \), and \( 2\pi \).

EXERCISES
Find the two square roots of each number. If the number is not a perfect square, approximate the values to the nearest 0.05.

1. \( \sqrt{16} \)
2. \( \frac{4}{25} \)
3. \( \sqrt{225} \)
4. \( \frac{1}{49} \)
5. \( \sqrt{10} \)
6. \( \sqrt{18} \)

Write all names that apply to each number.

7. \( \frac{7}{3} \)
   rational, real

8. \( -\sqrt{100} \)
   integer, rational, real

9. \( \frac{15}{5} \)
   whole, integer, rational, real

10. \( \sqrt{21} \)
    irrational, real

Compare. Write <, >, or =. (Lesson 1.3)

11. \( \sqrt{7} + 5 \) < \( 7 + \sqrt{5} \)
12. \( 6 + \sqrt{8} \) < \( \sqrt{6} + 8 \)
13. \( \sqrt{4} - 2 \) < \( 4 - \sqrt{2} \)

Order the numbers from least to greatest. (Lesson 1.3)

14. \( 8\sqrt{1}, \frac{32}{7}, 8.9 \)
15. \( 7, 2.55, \frac{7}{3}, 2.55, \sqrt{7} \)
1. An astronomer is studying Proxima Centauri, which is the closest star to our Sun. Proxima Centauri is approximately 4.9 billion kilometers away.

   a. Write this distance in scientific notation.

   b. Light travels at a speed of $3.0 \times 10^8$ m/s. How can you use this information to calculate the time in seconds it takes for light from Proxima Centauri to reach Earth? How many seconds does it take? Write your answer in scientific notation.

   c. Knowing that 1 year = $3.1536 \times 10^7$ seconds, how many years does it take for light to travel from Proxima Centauri to Earth? Write your answer in standard notation. Round your answer to two decimal places.

2. Cory is making a poster of common geometric shapes. He draws a square with a side of length $4\sqrt{3}$ cm, an equilateral triangle with a height of $\sqrt{200}$ cm, a circle with a circumference of $8\pi$ cm, a rectangle with length $\frac{122}{5}$ cm, and a parallelogram with base $3.14$ cm.

   a. Which of these numbers are irrational?

   b. Write the numbers in this problem in order from least to greatest. Approximate $\pi$ as $3.14$.

   c. Explain why $3.14$ is rational, but $\pi$ is not.
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**c.4.E** Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

**MODULE 3 Proportional Relationships**

**8.4.B, 8.4.C, 8.5.A, 8.5.E**

**Key Concepts**
- Proportional relationships or direct variation relationships can be described by an equation of the form $y = kx$, where $k$ is the constant of proportionality or the constant of variation. *(Lessons 3.1, 3.4)*
- A rate of change is the ratio of the amount of change in the output to the amount of change in the input. *(Lesson 3.2)*
- A relationship with a constant rate of change forms a line, and the rate of change is the slope of the line. *(Lesson 3.2)*
- The unit rate and the constant of proportionality are the same as the slope of a linear relationship. *(Lesson 3.3)*

**MODULE 4 Nonproportional Relationships**

**8.4.C, 8.5.B, 8.5.F, 8.9**

**Key Concepts**
- Linear relationships can be written in the slope-intercept form, $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept. When $b \neq 0$, the relationship between the variables is nonproportional. *(Lessons 4.1, 4.2, 4.3)*
- The $y$-intercept is the $y$-coordinate of the point where the graph intersects the $y$-axis. *(Lesson 4.2)*
- In a linear relationship $y = mx + b$, when $b \neq 0$, the relationship between the variables is nonproportional. *(Lessons 4.1, 4.4)*
- To solve a linear system of equations, a set of equations that have the same variables, find the point of intersection of the lines. *(Lesson 4.5)*
KEY CONCEPTS

• The equation of a linear relationship can be written in slope-intercept form if you know the slope and the y-intercept. (Lessons 5.1, 5.2, 5.3)

• A set of data made up of two paired variables is bivariate data and can have a linear or a nonlinear relationship. (Lesson 5.3)

• Bivariate data has a linear relationship if the rate of change is constant. Bivariate data with a nonlinear relationship will not have a constant rate of change. (Lessons 5.3)
MODULE 6 Functions

Key Concepts

• A function is a rule that assigns exactly one output to each input. (Lesson 6.1)
• Non-vertical lines are linear functions. All linear equations in the form $y = mx + b$ are linear functions. (Lesson 6.2)
• Two functions can be compared by comparing the slopes and y-intercepts. (Lesson 6.3)
**EXAMPLE 1**
Write an equation that represents the proportional relationship shown in the graph.

Use the points on the graph to make a table.

<table>
<thead>
<tr>
<th>Bracelets sold</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($)</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
</tbody>
</table>

Let $x$ represent the number of bracelets sold.
Let $y$ represent the profit.
The equation is $y = 3x$.

**EXAMPLE 2**
Find the slope of the line.

slope $= \frac{\text{rise}}{\text{run}}$
$= \frac{3}{2}$
$= 1.5$

**Key Vocabulary**
- constant of proportionality (constante de proporcionalidad)
- constant of variation (constante de variación)
- direct variation (variación directa)
- proportional relationship (relación proporcional)
- slope (pendente)

**EXERCISES**
1. The table represents a proportional relationship. Write an equation that describes the relationship. Then graph the relationship represented by the data. (Lessons 3.1, 3.3, 3.4)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (y)</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

$y = \frac{3}{2}x$

Find the slope and the unit rate represented on each graph. (Lesson 3.2)

2. Words
slope = unit rate = 2 words/min

3. Feet
slope = unit rate = 0.4 ft/s

**MODULE 4**
Nonproportional Relationships

**ESSENTIAL QUESTION**
How can you use nonproportional relationships to solve real-world problems?

**EXAMPLE 1**
Jai is saving to buy his mother a birthday gift. Each week, he saves $5. He started with $25. The equation $y = 5x + 25$ gives the total Jai has saved, $y$, after $x$ weeks. Draw a graph of the equation. Then describe the relationship.

Use the equation to make a table. Then, graph the ordered pairs from the table, and draw a line through the points.

<table>
<thead>
<tr>
<th>$x$ (weeks)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (savings in dollars)</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

The relationship is linear but nonproportional.
EXAMPLE 2

Graph \( y = \frac{1}{2}x - 2 \).
The slope is \( \frac{1}{2} \), or \( 0.5 \).
The \( y \)-intercept is \(-2\).

EXAMPLE 3

Solve the system of equations by graphing:
\[
\begin{align*}
y &= \frac{3}{4}x - 2 \\
y &= -x + 5
\end{align*}
\]
The point of intersection appears to be (4, 1). Check by substitution.
\[
\begin{align*}
y = \frac{3}{4}(4) - 2 &= 1 - 2 = -1 \\
y = -4 + 5 &= 1
\end{align*}
\]

EXERCISES

Complete each table. Explain whether the relationship between \( x \) and \( y \) is proportional or nonproportional and whether it is linear. (Lesson 4.1)

1. \( y = 10x - 4 \)

\[
\begin{array}{c|c|c|c|c|}
 x & 0 & 2 & 4 & 6 \\
 y & -4 & 16 & 36 & 56 \\
\end{array}
\]

2. \( y = -\frac{3}{2}x \)

\[
\begin{array}{c|c|c|c|c|}
 x & 0 & 1 & 2 & 3 \\
 y & 0 & -1.5 & -3 & -4.5 \\
\end{array}
\]

3. Find the slope and \( y \)-intercept for the linear relationship shown in the table. Graph the line. Is the relationship proportional or nonproportional? (Lessons 4.2, 4.4)

\[
\begin{array}{c|c|c|c|c|c|}
 x & -2 & -1 & 0 & 1 & 2 & 4 & 6 \\
 y & 0 & 2 & 4 & 6 & 8 & 10 \\
\end{array}
\]

slope __________ x-intercept ________

The relationship is proportional.

4. Tom’s Taxis charges a fixed rate of $4 per ride plus $0.50 per mile. Carla’s Cabs does not charge a fixed rate but charges $1.00 per mile. (Lessons 4.3, 4.5)

a. Write an equation that represents the cost of Tom’s Taxis.
   \[ y = 0.5x + 4 \]

b. Write an equation that represents the cost of Carla’s Cabs.
   \[ y = x \]

c. Steve calculated that for the distance he needs to travel, Tom’s Taxis will charge the same amount as Carla’s Cabs. Graph both equations. How far is Steve going to travel and how much will he pay?

Steve is going to travel 8 miles and will pay $8.00.

MODULE 5 Writing Linear Equations

EXERCISES

1. Jose is renting a backhoe for a construction job. The rental charge for a month is based on the number of days in the month and a set charge per month. In August, which has 31 days, Jose paid $700. In September, which has 30 days, Jose paid $715. Write an equation in slope-intercept form that represents this situation.

   \[
   y = mx + b
   \]
   \( x \), \( y \) \( x \), \( y \) \( x \), \( y \)

   Write the information given as ordered pairs.

   Find the slope.

   Slope-intercept form

   Substitute for \( y \), \( m \), and \( x \) to find \( b \).

   Solve for \( b \).

   Write the equation.
Does each of the following graphs represent a linear relationship? Why or why not? (Lesson 5.3)

6. Yes; the rate of change is \(-1\) between all the points.

7. No; the rates of change are different between different points.

**EXAMPLE 2**

Determine if the graph shown represents a linear or nonlinear relationship.

The rates of change are not constant. The graph represents a nonlinear relationship.

**EXERCISES**

1. Ms. Thompson is grading math tests. She is giving everyone that took the test a 10-point bonus. Each correct answer is worth 5 points. Write an equation in slope-intercept form that represents the scores on the tests. (Lesson 5.1)

   \[ y = 5x + 10 \]

The table shows a pay scale based on years of experience. (Lessons 5.1, 5.2)

<table>
<thead>
<tr>
<th>Experience (yr), ( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hourly pay ($), ( y )</td>
<td>9</td>
<td>14</td>
<td>19</td>
<td>24</td>
<td>29</td>
</tr>
</tbody>
</table>

2. Find the slope for this relationship. \( \frac{5}{2} \)

3. Find the \( y \)-intercept. \( 9 \)

4. Write an equation in slope-intercept form that represents this relationship. \( y = \frac{5}{2}x + 9 \)

5. Graph the equation, and use it to predict the hourly pay of someone with 10 years of experience.

Someone with 10 years of experience will be paid $34 an hour.

**EXAMPLE 1**

Determine whether each relationship is a function.

The relationship is not a function, because an input, 4, is paired with 2 different outputs, 4 and 0.

Since each input value is paired with only one output value, the relationship is a function.

\[ \begin{array}{|c|c|}
\hline
\text{Input} & \text{Output} \\
\hline
3 & 10 \\
4 & 4 \\
5 & 2 \\
4 & 0 \\
6 & 5 \\
\hline
\end{array} \]
**EXAMPLE 2**

Sally and Louis are on a long-distance bike ride. Sally bikes at a steady rate of 18 miles per hour. The distance y that Sally covers in x hours is given by the equation \( y = 18x \). Louis's speed can be found by using the numbers in the table. Who will travel farther in 4 hours and by how much?

<table>
<thead>
<tr>
<th>Louis's Biking Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h), x</td>
</tr>
<tr>
<td>Distance (mi), y</td>
</tr>
</tbody>
</table>

Sally's ride: 
\[ y = 18x \]
\[ y = 18(4) \]
\[ y = 72 \]

Louis's ride: 
\[ y = 20x \]
\[ y = 20(4) \]
\[ y = 80 \]

Sally will ride 72 miles in 4 hours. Louis will ride 80 miles in 4 hours. Louis will go 8 miles farther in 4 hours.

Each distance in the table is 20 times each number of hours. Louis's speed is 20 miles per hour, and his distance covered is represented by \( y = 20x \).

**EXERCISES**

Determine whether each relationship is a function. (Lesson 6.1)

1. function
2. function

Tell whether the function is linear or nonlinear. (Lesson 6.2)

3. \( y = 5x + \frac{1}{2} \) linear
4. \( y = x^2 + 3 \) nonlinear

5. Elaine has a choice of two health club memberships. The first membership option is to pay $500 now and then pay $150 per month. The second option is shown in the table. Elaine plans to go to the club for 12 months. Which option is cheaper? Explain.

<table>
<thead>
<tr>
<th>Months, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total paid ($), y</td>
<td>215</td>
<td>430</td>
<td>645</td>
</tr>
</tbody>
</table>

The first option is cheaper. $500 + 12($150) = $2300 is less than 12($215) = $2580.
Study Guide Review

Vocabulary Development

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**MODULE 7 Angle Relationships in Parallel Lines and Triangles**

**8.3.A, 8.4.A, 8.8.D**

**Key Concepts**

- The alternate interior angles formed by transversal intersecting parallel lines are congruent as are the alternate exterior angles. Same–side interior angles are supplementary. *(Lesson 7.1)*
- The sum of the interior angle measures of a triangle is 180°. *(Lesson 7.2)*
- The measure of the exterior angle of a triangle is equal to the sum of its remote interior angles. *(Lesson 7.2)*
- If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. *(Lesson 7.3)*

**MODULE 8 The Pythagorean Theorem**

**8.6.C, 8.7.C, 8.7.D**

**Key Concepts**

- If a triangle is a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. *(Lesson 8.1)*
- If the sum of the squares of the lengths of the legs of a triangle is equal to the square of the hypotenuse, then it is a right triangle. *(Lesson 8.2)*
- The Distance Formula states that the distance \(d\) between two points \((x_1, y_1)\) and \((x_2, y_2)\) is: \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\) *(Lesson 8.3)*
MODULE 9 Volume

Key Concepts
- A cylinder is a three-dimensional figure with 2 congruent circular bases. To find the volume of a cylinder, use the formula $V = Bh$ or $V = \pi r^2 h$. (Lesson 9.1)
- A cone is a three-dimensional figure with 1 circular base and 1 vertex. To find the volume of a cone, use the formula $V = \frac{1}{3} Bh$ or $V = \frac{1}{3} \pi r^2 h$. (Lesson 9.2)
- A sphere is a three-dimensional figure with all points the same distance from the center. To find the volume of a cylinder, use the formula $V = \frac{4}{3} \pi r^3$. (Lesson 9.3)
UNIT 3 Study Guide Review

Module 7 Angle Relationships in Parallel Lines and Triangles

Essential Question
How can you solve real-world problems that involve angle relationships in parallel lines and triangles?

Example 1
Find each angle measure when m∠6 = 81°.

A. m∠5 = 180° − 81° = 99°
   5 and 6 are supplementary angles.

B. m∠1 = 99°
   1 and 5 are corresponding angles.

C. m∠3 = 180° − 81° = 99°
   5 and 6 are same-side interior angles.

Example 2
Are the triangles similar? Explain your answer.

y = 180° − (67° + 35°)
y = 78°
x = 180° − (67° + 67°)
x = 46°

The triangles are not similar, because they do not have 2 or more pairs of corresponding congruent angles.

EXERCISES
1. If m∠GHA = 76°, find the measures of the given angles. (Lesson 7.1)
   m∠EGC = 76°
   m∠EGD = 104°
   m∠BHF = 76°
   m∠HGD = 76°

2. Find the measure of the missing angles. (Lesson 7.2)
   m∠JKM = 165°
   m∠LKM = 15°

3. Is the larger triangle similar to the smaller triangle? Explain your answer. (Lesson 7.3)
   The triangles are similar because all of their angles are congruent.

4. Find the value of x and y in the figure. (Lesson 7.3)
x = 10.5 cm and y = 12.5 cm

Module 8 The Pythagorean Theorem

Essential Question
How can you use the Pythagorean Theorem to solve real-world problems?

Example 1
Find the missing side length. Round your answer to the nearest tenth.

\[ a^2 + b^2 = c^2 \]
\[ 7^2 + b^2 = 18^2 \]
\[ 49 + b^2 = 324 \]
\[ b^2 = 275 \]
\[ b = \sqrt{275} \approx 16.6 \]

The length of the leg is about 16.6 inches.

Key Vocabulary
- hypotenuse (hipotenusa)
- leg (cateto)
- Pythagorean Theorem (teorema de Pitágoras)
EXAMPLE 2
Thomas drew a diagram to represent the location of his house, the school, and his friend Manuel’s house. What is the distance from the school to Manuel’s house? Round your answer to the nearest tenth.

EXERCISES
Find the missing side lengths. Round your answers to the nearest hundredth. (Lesson 8.1)

1. 

2. 

3. Hye Sun has a modern coffee table whose top is a triangle with the following side lengths: 8 feet, 3 feet, and 5 feet. Is Hye Sun’s coffee table top a right triangle? (Lesson 8.2)

No, it is not a right triangle.

4. Find the length of each side of triangle \( ABC \). If necessary, round your answers to the nearest hundredth. (Lesson 8.3)

\( AB \) 3 units

\( AC \) \( \sqrt{45} \approx 6.71 \) units

\( BC \) 6 units
MODULE 10  Surface Area

Key Concepts

- The surface area of a three-dimensional figure is the sum of the areas of all the faces of the figure, or the sum of the bases and the lateral area, \( S = 2B + L \). The formula is also written \( S = 2B + Ph \) for a prism, or \( S = 2\pi r^2 + 2\pi rh \) for a cylinder. (*Lessons 10.1, 10.2*)

Hydrologist

In Performance Task Item 1, students can see how a hydrologist uses mathematics on the job.

**SCORING GUIDES FOR PERFORMANCE TASKS**

1. **MATHEMATICAL PROCESSES**
   - Task Possible Points (Total: 6)
     - a 1 point for explanation: Multiply the density, \( 1000 \text{ kg/m}^3 \) by the volume of the cylinder, \( \pi r^2h \), to find the mass.
     - 2 points for correct calculation and mass: 26,533,000 kg
     - b 1 point for correct answer: No
     - 2 points for explanation: The volume of the aquifer is 34,618.5 \( \text{m}^3 \), which is 34,618,500 kg of water. Since the answer in a is less, the aquifer is not completely full.

2. **MATHEMATICAL PROCESSES**
   - Task Possible Points (Total: 6)
     - a 3 points for correct diagram:
     - b 2 points for correctly calculating that the total distance was 24 blocks.
     - 1 point for correctly calculating that the walk before the rain was 4 blocks longer than the walk home.
**Unit 3 Performance Tasks**

1. **Hydrologist**
   - A hydrologist needs to estimate the mass of water in an underground aquifer, which is roughly cylindrical in shape.
   - The diameter of the aquifer is 65 meters, and its depth is 8 meters.
   - One cubic meter of water has a mass of about 1000 kilograms.
   - a. The aquifer is completely filled with water. What is the total mass of the water in the aquifer? Explain how you found your answer.
     - Use $\pi \approx 3.14$ and round your answer to the nearest kilogram.
   - b. Another cylindrical aquifer has a diameter of 70 meters and a depth of 9 meters. The mass of the water in it is $27 \times 10^7$ kilograms.
     - Is the aquifer totally filled with water? Explain your reasoning.

2. **From his home, Myles walked his dog north 5 blocks, east 2 blocks, and then stopped at a drinking fountain. He then walked north 3 more blocks and east 4 more blocks. It started to rain so he cut through a field and walked straight home.**
   - a. Draw a diagram of his path.
   - b. How many blocks did Myles walk in all? How much longer was his walk before it started to rain than his walk home?
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MODULE 11 Equations and Inequalities with the Variable on Both Sides

Key Concepts
- To solve an equation or inequality with the same variable on both sides of the equal sign, add or subtract to eliminate the variable term from one side of the equation or inequality. (Lessons 11.1, 11.3)
- To eliminate fractions from an equation or inequality, multiply every term by the least common multiple of the denominators to create an equivalent equation or inequality. (Lessons 11.2, 11.4)
- To eliminate decimals from an equation or inequality, multiply every term by a power of 10 to eliminate the decimals and create an equivalent equation or inequality. (Lessons 11.2, 11.4)
**ESSENTIAL QUESTION**

How can you use equations and inequalities with variables on both sides to solve real-world problems?

**EXAMPLE 1**

A tutor gives students a choice of how to pay: a base rate of $20 plus $8 per hour, or a set rate of $13 per hour. Find the number of hours of tutoring for which the cost is the same for either choice.

Plan 1 cost: $20 + 8x
Plan 2 cost: $13x

Write the equation.

20 + 8x = 13x

Subtract 8x from both sides.

8x = 5x

Divide both sides by 5.

x = 4

The cost is the same for 4 hours of tutoring.

**EXAMPLE 2**

Solve 0.6y – 12.3 < 7.4 – 1.9y.

0.6y – 12.3 < 7.4 – 1.9y
Write the inequality.

10(0.6y – 10(12.3)) < 10(7.4 – 10(1.9y))
Multiply both sides by 10.

6y < 123 + 74 – 19y
Add 123 to both sides.

6y < 197 – 19y

19y < 197

y < 7.88
Divide both sides by 25.

**EXERCISES**

Solve. (Lessons 11.1, 11.2, 11.3, 11.4)

1. 13 – 6y = 8y
   y = 0.93

2. \(\frac{1}{2}x + 5 = \frac{1}{3}x\)
   x = 20

3. 7.3x + 22 ≤ 2.1x – 22.2
   t = –8.5

4. \(7 - 45z < 5z + 13\)
   z > –0.12

5. \(\frac{1}{2} + z \geq \frac{1}{3} - 0.8\)
   e ≥ –11

6. \(0.75x - 6.5 - 0.5 - 0.25x\)
   x = 6

7. Write a real-world situation that could be modeled by the equation 650 = 10m + 60m + 400. (Lesson 11.1)
   Sample answer: Jill and Sam are both putting money in their savings accounts. Jill starts with $650 and puts in $10 a month. Sam starts with $400 and puts in $60 a month. After how many months will Jill and Sam have the same amount in their accounts?

8. John is trying to decide which carpeting company to use to put carpet in his living room. Carla’s Carpeting charges $45 plus $5.25 per square foot. Fred’s Flooring charges $195 plus $4.25 per square foot. For what size room is Carla’s Carpeting cheaper than Fred’s Flooring? (Lesson 11.3)
   Carla’s is cheaper for rooms with less than 120 square feet.
Study Guide Review

Integrating the ELPS
Encourage English learners to refer to their notes and the illustrated, bilingual glossary as they review the unit content.

ELPS c.4.E Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

MODULE 12 Transformations and Congruence

Key Concepts
- A transformation is a function that changes the position, size, or shape of a figure. (Lesson 12.1)
- Translations, reflections, and rotations are transformations that preserve the size and shape of the preimage. (Lessons 12.1, 12.2, 12.3)
- A translation is a transformation that slides a figure along a straight line. (Lesson 12.1)
- A reflection is a transformation that flips a figure across a line. Each point and its image are the same distance from the line of reflection. (Lesson 12.2)
- A rotation is a transformation that turns a figure around a point called the center of rotation. (Lesson 12.3)
- To reflect an image over the x-axis, change the sign of the y-coordinates, and to reflect an image over the y-axis, change the sign of the x-coordinates. (Lesson 12.4)
MODULE 13  Dilations, Similarity, and Proportionality

8.3.A, 8.3.B, 8.3.C, 8.10.A, 8.10.B, 8.10.D

Key Concepts

- A dilation is a transformation that changes the position and size but not the shape of a figure. *(Lesson 13.1)*
- The scale factor of a dilation describes how much the figure is enlarged or reduced and is the ratio of a length of the image to the corresponding length of the preimage. *(Lesson 13.1)*
- To find the coordinates of a dilated image with the origin as the center, multiply the x- and y-coordinates of the vertices by the scale factor. *(Lesson 13.2)*
- The side length and perimeter of a dilated figure is the corresponding measurement of the original figure multiplied by the scale factor. The area of the dilated figure is the area of the original figure multiplied by the square of the scale factor. *(Lesson 13.3)*
EXAMPLE
Translate triangle \(\triangle XYZ\) left 4 units and down 2 units. Graph the preimage and label the vertices.

 Translate the vertices by subtracting 4 from each \(x\)-coordinate and 2 from each \(y\)-coordinate. The new vertices are \(X'(-1, 1)\), \(Y'(0, 3)\), and \(Z'(1, -3)\).

 Connect the vertices to draw \(\triangle XYZ'\).

EXERCISES
Perform the transformation shown. (Lessons 12.1, 12.2, 12.3)

1. Reflection over the \(x\)-axis

2. Translation 5 units right

3. Rotation \(90^\circ\) counterclockwise about the origin

4. Translation 4 units right and 4 units down

5. Quadrilateral \(ABCD\) with vertices \(A(4, 4)\), \(B(5, 1)\), \(C(5, -1)\) and \(D(4, -2)\) is translated left 2 units and down 3 units. Graph the preimage and the image. (Lesson 12.4)

6. Triangle \(\triangle RST\) has vertices at \((-8, 2)\), \((-4, 0)\), and \((-12, 8)\). Find the vertices after the triangle has been reflected over the \(y\)-axis. (Lesson 12.4)

\((-8, 2)\), \((4, 0)\), \((12, 8)\)

7. Triangle \(\triangle XYZ\) has vertices at \((3, 7)\), \((9, 14)\), and \((12, -1)\). Find the vertices after the triangle has been rotated \(180^\circ\) about the origin. (Lesson 12.4)

\((-3, -7)\), \((-9, -14)\), \((-12, 1)\)
**Dilations, Similarity, and Proportionality**

**ESSENTIAL QUESTION**
How can you use dilations, similarity, and proportionality to solve real-world problems?

**EXAMPLE**
Dilate triangle ABC with the origin as the center of dilation and scale factor \( \frac{1}{2} \). Graph the dilated image.

Multiply each coordinate of the vertices of ABC by \( \frac{1}{2} \) to find the vertices of the dilated image.
- A(5, 1) \( \rightarrow \) A'(2.5, -0.5)
- B(4, -5) \( \rightarrow \) B'(2, -2.5)
- C(2, 0) \( \rightarrow \) C'(1, 0)

**EXERCISES**

1. For each pair of corresponding vertices, find the ratio of the x-coordinates and the ratio of the y-coordinates. (Lesson 13.1)

<table>
<thead>
<tr>
<th>Ratio of x-coordinates:</th>
<th>Ratio of y-coordinates:</th>
<th>Scale factor of the dilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

2. Andrew’s old television had a width of 32 inches and a height of 18 inches. His new television is larger by a scale factor of 2.5. Find the perimeter and area of Andrew’s old television and his new television. (Lesson 13.3)

<table>
<thead>
<tr>
<th>Perimeter of old TV:</th>
<th>100 in.</th>
<th>Perimeter of new TV:</th>
<th>250 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of old TV:</td>
<td>576 in²</td>
<td>Area of new TV:</td>
<td>3,600 in²</td>
</tr>
</tbody>
</table>

**Key Vocabulary**
- center of dilation (centro de dilatación)
- dilation (dilatación)
- enlargement (aumento)
- reduction (reducción)
- scale factor (factor de escala)

**Dilate each figure with the origin as the center of the dilation. List the vertices of the dilated figure then graph the figure. (Lesson 13.2)**

3. (x, y) \( \rightarrow \) \( \left( \frac{1}{2}x, \frac{3}{2}y \right) \)
   - X'(−2, 1); Y'(−1, 1); Z'(1, 2)

4. (x, y) \( \rightarrow \) \( (2x, 2y) \)
   - A'(-2, 4); B'(4, 4); C'(6, -2); D'(0, -2)
Study Guide Review

Integrating the ELPS
Encourage English learners to refer to their notes and the illustrated, bilingual glossary as they review the unit content.

• **c.4.E** Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

MODULE 14  Scatter Plots

**8.5.C, 8.5.D, 8.5.I, 8.11.A**

**Key Concepts**

- A scatter plot is a graph with points plotted to show the relationship between two sets of data. *(Lesson 14.1)*
- If two sets of data increase together, they show a positive association. If one set increases while the other decreases, they have a negative association. If changes in one data set have no effect on the other, they have no association. *(Lesson 14.1)*
- Data that have a linear association cluster along a line and can be modeled by a trend line. *(Lessons 14.1, 14.2)*
MODULE 15  Sampling

**Key Concepts**

- The mean is a measure that describes the center of data and is found by dividing the sum of the data values by the number of values. *(Lesson 15.1)*
- The mean absolute deviation is a measure of variability that describes the spread of data and is found by finding the mean of the distance each value is from the mean of the data set. *(Lesson 15.1)*
- Information can be gathered about a population by analyzing a sample, or part of the population. A random sample is the best representation of a population. *(Lesson 15.2)*
**Key Vocabulary**
- cluster (agrupación)
- outlier (valor extremo)
- scatter plot (diagrama de dispersión)
- trend line (línea de tendencia)

**EXERCISES**

1. The table shows the income of 8 households, in thousands of dollars, and the number of televisions in each household. (Lesson 14.1)

<table>
<thead>
<tr>
<th>Income ($1000)</th>
<th>20</th>
<th>20</th>
<th>30</th>
<th>30</th>
<th>40</th>
<th>60</th>
<th>70</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of televisions</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data.
b. Describe the association between income and number of televisions. Are any of the values outliers? The data generally shows a positive association between income and number of televisions. As income increases, so does the number of televisions. There is an outlier at (20, 4).

c. When the price of the product is $3.50, the number of potential buyers will be about **Sample answer: 17**.
d. When the price of the product is $5.50, the number of potential buyers will be about **Sample answer: 13**.

2. The scatter plot shows the relationship between the price of a product and the number of potential buyers. (Lesson 14.2)
a. Draw a trend line for the scatter plot.
b. Write an equation for your trend line. **Sample answer: \( y = -2x + 24 \)**
c. When the price of the product is $3.50, the number of potential buyers will be about **Sample answer: 17**.
d. When the price of the product is $5.50, the number of potential buyers will be about **Sample answer: 13**.
Sampling

**Essential Question**
How can you use sampling to solve real-world problems?

**Example**
Robert is a waiter. He wants to earn an average of $85 or more in tips each night with no more than $10 variability. His earnings in tips per night for 10 nights are shown in the table. Is Robert meeting his goals?

<table>
<thead>
<tr>
<th>Robert's Tips per Night</th>
<th>$92</th>
<th>$70</th>
<th>$105</th>
<th>$89</th>
<th>$90</th>
<th>$110</th>
<th>$72</th>
<th>$71</th>
<th>$98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean absolute deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On average, Robert is making $88.70 in tips per night, which is more than $85. His earnings have a mean absolute deviation of $10.62, which is higher than $10. He is not meeting his goals.

**Exercises**
1. Find the mean and mean absolute deviation of the set of data. Round to the nearest hundredth. (Lesson 15.1)

   **Distance per day (mi) driven by Juan**
   
<table>
<thead>
<tr>
<th>12</th>
<th>9</th>
<th>7</th>
<th>7</th>
<th>11</th>
<th>10</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean: 9 mi</td>
<td>Mean absolute deviation: 12/7 = 1.7 mi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. A pottery store gets a shipment of 1500 dishes and wants to estimate how many dishes are broken. The manager will use a random sample to represent the entire shipment. In actuality, 18% of the dishes are broken. (Lesson 15.2)

   You will simulate the manager's test by generating random numbers between 1 and 1500. Explain what the generated numbers will mean.

   **The numbers 1–270 will represent the broken dishes, and 271–1500 will represent the unbroken dishes.**

   Use the graphing calculator function randInt(1,1500) to generate 30 numbers.

   According to the sample, how many broken dishes should the manager expect to find in the shipment?

   **Answers will vary.**
**Study Guide Review**

**Integrating the ELPS**
Encourage English learners to refer to their notes and the illustrated, bilingual glossary as they review the unit content.

**Additional Resources**
- Personal Math Trainer
- Online Assessment and Intervention
- Assessment Resources
  - Leveled Unit Tests: A, B, C, D
  - Performance Assessment
  - End-of-Year Test Modules 11–16

**MODULE 16 Managing Your Money and Planning for Your Future**

**Key Concepts**
- There are two important factors to consider when borrowing money, the interest rate and the time it will take to pay off the total amount of the loan. *(Lesson 16.1)*
- To calculate simple interest, use the formula \( I = Prt \). To calculate compound interest, use the formula \( A = P(1 + rt) \). *(Lesson 16.2)*
- There are several ways to pay for goods and services. Making purchases and other financial decisions can be done responsibly or irresponsibly. Factors to consider include income, savings, debt, and interest. *(Lesson 16.3)*
- When estimating the cost of a college education, there are many factors to consider, such as if the college is in-state or out-of-state, the cost of room and board, the cost of textbooks, and more. Tools such as online college estimation websites can help you estimate the cost of your college education. *(Lesson 16.4)*

**TEKS**

**ELPS**
- c.4.E Read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.
Managing Your Money and Planning for Your Future

How can you manage your money and plan for a successful financial future?

**EXAMPLE 1**
Clayton has $5,000 in an account earning simple interest at a rate of 2.5% per year. His wife Candice has $5,000 in an account earning interest at a rate of 2.3% compounded annually. How much interest did each account earn over 15 years? Which account is worth more after 15 years?

Clayton - Simple Interest:

\[ I = Prt \]

\[ A = P(1 + rt) \]

Candice - Compound Interest:

\[ I = \frac{P(1 - (1 + r)^{-t}}{1} \]

\[ A = P(1 + r)^t \]

Clayton earned $1,875 in interest over 15 years for a total of $6,875 in his savings account.

Candice earned $7,032.42 – $5,000 = $2,032.42 in interest for a total of $7,032.42 in her account. Candice earned more interest and has more money in her savings account.

**EXAMPLE 2**
Lee earns an annual salary of $42,000. He has $2,300 in savings and $1,500 in credit card debt. Lee finances a new truck with monthly payments of $525 and a down payment of $1,300 that he takes out of savings. Was Lee’s decision financially responsible or financially irresponsible?

Lee made a financially irresponsible decision.

Reason 1: If Lee loses his job, he does not have enough savings to cover his car payments for more than 4 months.

Reason 2: Lee could have continued driving his current truck and used the money in savings to pay off his credit card debt.

**EXERCISES**

1. Sheri is going to take out a loan for $4,000 that she plans to pay back in 2 years. She wants to know how much more it will cost her in interest if she uses her credit card at 18% interest instead of borrowing from the bank at 10% interest. Use an online calculator to find the total repayment for each loan and the difference in the cost of these two choices. (Lesson 16.1)

   Credit card: $4,792.80; bank: $4,429.92. Sheri will pay $362.88 more if she uses her credit card.

2. You are trying to decide which account to put $3,500 into for the next 6 years. One account has an interest rate of 2.9% and is compounded annually. The other account has a simple interest rate of 3.1%. Which account will earn more interest over 6 years, and how much more interest will it earn? (Lesson 16.2)

   The savings account with the compound interest will earn $3.89 more.

3. Mana has $120 to spend on food for the week. She goes out to a restaurant to eat dinner with her friends and spends $62 on the meal. Did Maria make a financially responsible decision or a financially irresponsible decision? Explain your answer. (Lesson 16.3)

   Sample answer: Maria spent more than 50% of her food budget on one meal. She made a financially irresponsible decision.

4. Use an online tool to estimate the cost for one year at a 4-year university and one year at a 2-year college in Texas. (Lesson 16.4)

   **Answers will vary.**

<table>
<thead>
<tr>
<th>4-year university</th>
<th>2-year college</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuition &amp; Fees</td>
<td></td>
</tr>
<tr>
<td>Room &amp; Board</td>
<td></td>
</tr>
<tr>
<td>Books</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

   a. Find the cost of attending the university for four years. **Answers will vary but should be 4 times the one-year total.**

   b. Find the cost of attending the two-year college and transferring to the university for your final two years of school. **Answers will vary but should be 2 times the one-year total in column 2 added to 2 times the one-year total in column 1.**